Transverse Polarization of the K-Conversion Electrons following the Beta Decay of Hg^{203}†

R. A. LLEWELLYN* AND R. M. STEFFEN
Department of Physics, Purdue University, Lafayette, Indiana
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The transverse polarization of the K-conversion electrons following the β decay of Hg^{203} has been investigated. A lens spectrometer was used to focus the conversion electrons on a Mott-scattering foil and the right-left asymmetry of the scattered conversion electrons in coincidence with the preceding β particles was measured. A detailed discussion of the instrumental corrections is presented. The degree of transverse polarization of the Hg^{203} conversion electrons in the plane defined by the momenta $\mathbf{p}_\beta = m\mathbf{v}$ of the β particles and $\mathbf{p}_c$ of the conversion electrons is described by $P_{\text{II}}(\theta) = K(v/c)\sin\theta$, where θ is the angle between $\mathbf{p}_\beta$ and $\mathbf{p}_c$. The experiments yielded the value of the polarization constant $K = +0.62 \pm 0.08$. The positive sign of K implies that the spin of the conversion electrons is pointing in the same direction as the β-particle momentum $\mathbf{p}_\beta$. A comparison with the theory of Becker and Rose shows that the experimental value of K is compatible with the spin assignments $I_i = \frac{1}{2}$ and $I_i = \frac{3}{2}$ to the ground state of Hg^{203}. The spin $I_i = \frac{5}{2}$ is excluded. The most probable spin assignment is $I_i = \frac{3}{2}$, which gives good agreement with the observed polarization if the ratio y' of the scalar and the vector-type beta matrix elements is $y' = -1.25 \pm 0.15$.

1. INTRODUCTION

As a consequence of the nonconservation of parity in weak interactions which are responsible for β decay, the ensemble of nuclear states formed as a result of a β-decay process is polarized with respect to the direction of the observation of the β particle. Gamma radiation emitted from such a polarized ensemble of nuclei is circularly polarized and the measurement of the β direction—γ circular polarization correlation has become a very useful tool in the study of the degree of violation of parity conservation as well as in the study of the β-decay interaction in general. The measurement of the degree of circular polarization of gamma radiation, however, involves the detection of the circular-polarization-dependent asymmetry in the scattering of γ rays from polarized electrons (available, e.g., in magnetized iron), a process which gives rise to relatively small measurable effects (at best a few percent), even if the degree of circular polarization of the γ radiation is large. The effects are particularly small for γ energies below 0.300 MeV. In view of this difficulty, other methods of determining the state of polarization of the nuclear ensemble following β decay have been proposed. Following a suggestion by Frauenfelder *et al.*,[1] Rose and Becker[2] computed the degree of polarization of K-conversion electrons emitted from nuclear states that are formed in β decay. Independently, Geshkenbein,[3] and Berestetskii and Rudik[4] studied the polarization of conversion electrons following β transitions.

These authors have shown that conversion electrons following β decay have an appreciable transverse polarization in the plane defined by the beta-particle momentum vector $\mathbf{p}_\beta$ and the conversion electron momentum vector $\mathbf{p}_c$, if the conversion electron is observed perpendicular to the direction of beta emission. Measurements of this transverse polarization of conversion electrons were first reported by Liubimov *et al.*[5] and by Alberghini and Steffen.[6] Both research groups determined the polarization of the K-conversion electrons following the β decay of Hg^{203}.

The results reported by the two groups were in agreement with respect to the magnitude of the polarization; however, the *direction* of the polarization vector reported by the two groups was different. Hence, it seemed desirable to repeat the measurement of the transverse polarization of the K-conversion electrons following the β decay of Hg^{203} with improved technique. During the course of this investigation, similar measurements on the Hg^{203} conversion electrons were reported by Jüngst and Schmicker.[7] An extensive study of the transverse polarization of conversion electrons following β decay including the Hg^{203} case has been made by Blake *et al.*[8] The former paper supports the result of Liubimov *et al.*[5]; the latter reports results on Hg^{203} in agreement with the results of Alberghini and Steffen.[6]

The results presented here and reported in a previous communication[9] confirm the polarization direction found by Alberghini and Steffen,[6] although the magnitude of the polarization was found to be somewhat larger than previously reported.

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* Present address: Physics Department, Rose Polytechnic Institute, Terre Haute, Indiana.
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2. THEORY

The observation of the momentum direction $\mathbf{p}_\beta$ of β particles emitted from a randomly oriented ensemble of nuclei leads to an ensemble of nuclear states whose orientation is axially symmetric about the measured direction. The state of orientation is described by a $(2I+1)\times(2I+1)$ density matrix $\rho_{mm'}$, where I is the quantum number of total angular momentum of the daughter nucleus. If the observation direction $\mathbf{p}_\beta$ of the β particles is chosen as quantization axis, the density matrix $\rho_{mm'}$ is diagonal and the diagonal elements ρ_{mm} may be interpreted as the *populations* of a pure state of magnetic quantum number m:

$$P(m)=\rho_{mm}=\sum_{k=0}^{2I}(2I+1)^{1/2}(-1)^{m+I}\times\begin{pmatrix} I & I & k\\ m & -m & 0\end{pmatrix}(2k+1)^{1/2}A_k'(\beta), \quad (1)$$

where $A_k'(\beta)$ is a parameter describing the β transition. In terms of the particle parameters $b(L,L')$ of the beta transition, as defined and tabulated, e.g., by Alder *et al.*,[10] the parameter $A_k'(\beta)$ is given by

$$A_k'(\beta)=\sum_{L,L'}F_k(LL'I_iI)b_k(LL'). \quad (2)$$

The F coefficients $F_k(LL'I_iI)$ which are important in all angular correlation problems are defined by

$$F_k(LL'I_iI) =(-1)^{I+I_i+1}[(2I+1)(2L+1)(2L'+1)(2k+1)]^{1/2}\times\begin{pmatrix} L & L' & k\\ 1 & -1 & 0\end{pmatrix}\begin{Bmatrix} I & I & k\\ L & L' & I_i\end{Bmatrix}. \quad (3)$$

The quantum number I_i describes the total angular momentum of the initial nuclear state. The multipole orders L give the total angular momentum carried away by the lepton radiation field.

If the β transition is followed by a gamma transition, the $\beta-\gamma$ angular correlation is given by[10]

$$W(\theta,\tau)=\sum_{k=0}^{k_{\max}}(-\tau)^kA_k'(\beta)A_k(\gamma)P_k(\cos\theta), \quad (4)$$

where θ is the angle between the β-particle momentum and the γ-ray momentum. The helicity τ of the γ radiation is $\tau=+1$ or $\tau=-1$ for right- or left-circular polarization. For a mixed gamma transition of multipole transition amplitudes $\langle I\|L_\gamma\|I_f\rangle$ and $\langle I\|L_\gamma'\|I_f\rangle$, the γ angular-correlation parameter $A_k'(\gamma)$ is

$$A_k'(\gamma)=F_k(L_\gamma L_\gamma I_fI)+2\delta(\gamma)F_k(L_\gamma L_\gamma'I_fI)+\delta^2(\gamma)F_k(L_\gamma'L_\gamma'I_fI), \quad (5)$$

where

$$\delta(\gamma)=\langle I\|L_\gamma'\|I_f\rangle/\langle I\|L_\gamma\|I_f\rangle. \quad (6)$$

The angular-momentum quantum number of the final nuclear state of the $\beta-\gamma$ cascade is denoted by I_f.

We define the normalized angular-correlation parameters $A_k(\gamma)$ and $A_k(\beta)$,

$$A_k(\gamma)=A_k'(\gamma)/A_0'(\gamma)=A_k'(\gamma)/(1+\delta^2), \quad (7)$$

$$A_k(\beta)=[A_k'(\beta)/A_0'(\beta)](-1)^k. \quad (8)$$

The $\beta-\gamma$ angular correlation involving allowed and first-forbidden β transitions can then be written in the form,[10,11]

$$W(\theta,\tau)=1+\tau A_1(\beta)A_1(\gamma)P_1(\cos\theta)+A_2(\beta)A_2(\gamma)P_2(\cos\theta)+\tau A_3(\beta)A_3(\gamma)P_3(\cos\theta). \quad (9)$$

For allowed β transitions, only $A_1(\beta)$ is nonvanishing if higher order effects (e.g., weak magnetism effects,[12] cross terms of allowed and second-forbidden β matrix elements[13]) are neglected. The β correlation parameter $A_1(\beta)$ is[10]

$$A_1(\beta)=\pm\frac{2}{3}\frac{F_1(10I_iI)y+F_1(11I_iI)y^2}{1+y^2}\frac{p}{W}\quad\text{for}\quad\beta^\mp, \quad (10)$$

where

$$y=C_A\int\boldsymbol{\sigma}\Big/C_V\int 1 \quad (11)$$

is the ratio of the Gamow-Teller to the Fermi component amplitude.

For first-forbidden β transitions, the parameters $A_k(\beta)$ are nonvanishing, in general, for $k\leq 3$. The $A_3(\beta)$ term, however, is negligible if the contributions of the tensor component, characterized by the nuclear matrix element $\int B_{ij}$ can be neglected in comparison to the contributions of the vector type ($\int\mathbf{r}$, $\int i\boldsymbol{\sigma}\times\mathbf{r}$, $\int i\boldsymbol{\alpha}$) and the scalar type ($\int i\gamma_5$, $\int\boldsymbol{\sigma}\cdot\mathbf{r}$) components.

If the ξ approximation is applicable to a first-forbidden β transition, i.e., if $\xi=\alpha Z/2R\gg W_0$, and if the contribution of the $\int B_{ij}$ component is small, the correlation factor $A_1(\beta)$ is also given by Eq. (10), where the matrix element parameter y is replaced by

$$y'=\frac{C_V\int i\boldsymbol{\alpha}-\xi\left(C_V\int\mathbf{r}-C_A\int i\boldsymbol{\sigma}\times\mathbf{r}\right)}{-C_A\int i\gamma_5-\xi C_A\int\boldsymbol{\sigma}\cdot\mathbf{r}}. \quad (12)$$

The degree of circular polarization $P_c(\theta)$ of gamma radiation observed at an angle θ with respect to the

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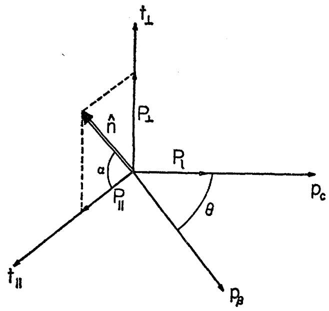

FIG. 1. Polarization vectors describing the state of polarization of conversion electrons emitted after β decay.

momentum direction of a preceding β particle is, according to Eq. (9),

$$P_c(\theta)=\frac{A_1(\beta)A_1(\gamma)P_1(\cos\theta)+A_3(\beta)A_3(\gamma)P_3(\cos\theta)}{1+A_2(\beta)A_2(\gamma)P_2(\cos\theta)}. \quad (13)$$

For an allowed transition this expression reduces to

$$P_c(\theta)=A_1(\beta)A_1(\gamma)\cos\theta. \quad (14)$$

For a nonunique β transition described by the ξ approximation,

$$P_c(\theta)_\xi=\frac{A_1(\beta)A_1(\gamma)\cos\theta}{1+A_2(\beta)A_2(\gamma)P_2(\cos\theta)}, \quad (15)$$

where $A_2(\beta)A_2(\gamma)$ is the anisotropy factor of the $\beta-\gamma$ directional correlation.

If the γ transition following the β decay is internally converted, the polarization state of the conversion electrons with respect to the momentum direction $\mathbf{p}_\beta$ of the β particles and with respect to the momentum direction $\mathbf{p}_c$ of the conversion electrons may be investigated.

We define the "polarization" vector of the conversion electron beam as the ensemble average of the relativistic polarization (vector) operator

$$\mathbf{P}=\langle\mathbf{P}_{\rm op}\rangle=\langle\boldsymbol{\sigma}\cdot\hat{p}_c\rangle\hat{p}_c+\beta\langle\boldsymbol{\sigma}\cdot\mathbf{t}_{\shortparallel}\rangle\mathbf{t}_{\shortparallel}+\beta\langle\boldsymbol{\sigma}\cdot\mathbf{t}_\perp\rangle\mathbf{t}_\perp, \quad (16)$$

where the unit vectors $\hat{p}_c$, $\mathbf{t}_\perp$, and $\mathbf{t}_{\shortparallel}$ form an orthogonal basis (Fig. 1). The unit vector $\mathbf{t}_{\shortparallel}$ is *in* the plane of $\mathbf{p}_\beta$ and $\mathbf{p}_c$ and $\mathbf{t}_\perp$ is normal to the plane of $\mathbf{p}_\beta$ and $\mathbf{p}_c$. If θ is the angle between $\mathbf{p}_\beta$ and $\mathbf{p}_c$,

$$\mathbf{t}_{\shortparallel}=\hat{p}_c\times(\hat{p}_\beta\times\hat{p}_c)/\sin\theta \quad (17)$$

and

$$\mathbf{t}_\perp=(\hat{p}_\beta\times\hat{p}_c)/\sin\theta. \quad (18)$$

The first term of Eq. (16) $\langle\boldsymbol{\sigma}\cdot\hat{p}_c\rangle$ describes the *longitudinal* polarization P_l (positive in the direction of $\mathbf{p}_c$), the second term $\beta\langle\boldsymbol{\sigma}\cdot\mathbf{t}_{\shortparallel}\rangle$ describes the *transverse* polarization $P_{\shortparallel}$ *in the plane* of $\mathbf{p}_\beta$ and $\mathbf{p}_c$ (positive polarization in the direction of the vector $\mathbf{t}_{\shortparallel}$), and the last term $\beta\langle\boldsymbol{\sigma}\cdot\mathbf{t}_\perp\rangle$ describes the *transverse* polarization $P_\perp$ *out of the plane* of $\mathbf{p}_\beta$ and $\mathbf{p}_c$ (positive polarization in the direction of $\mathbf{t}_\perp$). Thus, the polarization vector $\mathbf{P}(\theta)$ of the conversion electron beam may be expressed in the form

$$P(\theta)=P_l(\theta)\hat{p}_c+P_{\shortparallel}(\theta)\mathbf{t}_{\shortparallel}+P_\perp(\theta)\mathbf{t}_\perp. \quad (19)$$

For the discussion of experimental methods of measuring the polarization of an electron beam, it is convenient to write Eq. (19) as

$$\mathbf{P}(\theta)=P_l(\theta)\hat{p}_c+P_T(\theta)\mathbf{n}, \quad (20)$$

where the vector $\mathbf{n}$ is a unit vector in the direction of maximum transverse polarization:

$$\mathbf{n}=\cos\alpha\,\mathbf{t}_{\shortparallel}+\sin\alpha\,\mathbf{t}_\perp, \quad (21)$$

and where the angle α is determined by

$$\tan\alpha=P_\perp(\theta)/P_{\shortparallel}(\theta),\quad 0\leq\alpha<\pi. \quad (22)$$

We refer to the direction $\mathbf{n}$ as the transverse polarization axis. The scalar quantity $P_T(\theta)$,

$$P_T(\theta)=\pm[P_{\shortparallel}^2(\theta)+P_\perp^2(\theta)]^{1/2} \quad (23)$$

is referred to as simply the transverse polarization of the beam. Since we restrict the angle α to $0\leq\alpha<\pi$, the sense of direction into which the polarization vector points is determined by the sign of $P_T(\theta)$.

A remark may be added concerning the treatment of the polarization of an electron beam as an ordinary vector. Suppose we have an electron in the state with spin "up" with respect to some direction z; its state is described by

$$\chi_{1/2}{}^{1/2}(z)=\begin{pmatrix}1\\0\end{pmatrix}.$$

With respect to some other reference direction $\mathbf{n}$ at an angle α and φ with respect to z, its state is given by

$$\chi^{1/2}(\mathbf{n})=\begin{pmatrix}\cos(\frac{1}{2}\alpha)\\ e^{i\varphi}\sin(\frac{1}{2}\alpha)\end{pmatrix},$$

and the expectation value of its spin component along $\mathbf{n}$ is evidently given by $\cos^2(\frac{1}{2}\alpha)-\sin^2(\frac{1}{2}\alpha)=\cos\alpha$, which justifies our vector treatment.

The transverse polarization $P_T(\theta)$ may also be expressed in the following way. If the number of electrons in the beam with spin component "parallel" to $\mathbf{n}$ is $N_+(\theta)$ and the number with "antiparallel" spin is $N_-(\theta)$, the degree of transverse polarization is

$$P_T(\theta)=[N_+(\theta)-N_-(\theta)]/[N_+(\theta)+N_-(\theta)]. \quad (24)$$

The polarization of conversion electrons following β decay has been computed, e.g., by Rose and Becker.[2] The polarization components $P_l(\theta)$, $P_{\shortparallel}(\theta)$, and $P_\perp(\theta)$ of conversion electrons following a nonunique first-forbidden transition that is described by the ξ approximation are given by

$$P_l(\theta)=-\frac{A_1(\beta)C_{1,l}(e^-)\cos\theta}{1+A_2(\beta)C_{2,0}(e^-)P_2(\cos\theta)}, \quad (25)$$

$$P_{\shortparallel}(\theta)=-\frac{A_1(\beta)C_{1,\shortparallel}(e^-)\sin\theta}{1+A_2(\beta)C_{2,0}(e^-)P_2(\cos\theta)}, \quad (26)$$

$$P_\perp(\theta)=\frac{A_2(\beta)C_{2,\perp}(e^-)P_2'(\cos\theta)}{1+A_2(\beta)C_{2,0}(e^-)P_2(\cos\theta)}. \quad (27)$$

The polarization parameters $C_{1,l}(e^-)$, $C_{1,\|}(e^-)$, and $C_{1,\perp}(e^-)$ and the directional parameter $C_{2,0}(e^-)$ of the conversion electrons depend on Z and on the energy of the electromagnetic transition, as well as on the multipole and parity character of the transition. For a multipole transition that is a mixture of two components of multipole order L_c and L_c' with parities π and π', the conversion electron parameters are given by

$$\begin{aligned}C_{k,q}(e^-)=&F_k(L_cL_cI_fI)b_{k,q}(L_c,\pi)\\&+2\delta(e^-)F_k(L_cL_c'I_fI)b_{k,q}(L_c,L_c',\pi,\pi')\\&+\delta^2(e^-)F_k(L_c'L_c'I_fI)b_{k,q}(L_c',\pi'),\quad(28)\end{aligned}$$

where the index q stands for l, $\|$, $\perp$, or 0. The parameter $\delta(e^-)$ is the amplitude ratio of the two conversion electron multipole components (L_c,π) and $L_c',\pi')$. For K-conversion electrons,

$$\delta(e_K)=\frac{\langle I\|e_KL_c'\pi'\|I_f\rangle}{\langle I\|e_KL_c\pi\|I_f\rangle}=\left(\frac{\alpha_K(L_c',\pi')}{\alpha_K(L_c,\pi)}\right)^{1/2}\delta(\gamma),\quad(29)$$

where $\alpha_K(L_c\pi)$ is the coefficient of internal K conversion for the $(L_c\pi)$ multipole transition and $\delta(\gamma)$ is the γ-mixing ratio (6).

The particle parameters $b_{k,q}(e^-)$ for K-conversion electrons are tabulated in the paper of Rose and Becker.[2]

For even k, $b_{k,l}(L_c,\pi)$ and $b_{k,\|}(L_c,\pi)$ vanish; $b_{k\perp}(L_c,\pi)$ and $b_{k0}(L_c,\pi)$ vanish for odd k. This behavior is to be expected from general symmetry considerations. The polarization observables $P_l\propto\langle\boldsymbol{\sigma}\cdot\hat{p}_\beta\rangle$ and

$$P_\|\propto\langle\{\boldsymbol{\sigma}\cdot[\hat{p}_\beta\times(\hat{p}_c\times\hat{p}_\beta)]\}\rangle$$

are typical pseudoscalar observables, which give evidence of the violation of parity in the preceding β transition [$A_k(\beta)$ terms with odd k], whereas $P_\perp\propto\langle[\boldsymbol{\sigma}(\hat{p}_\beta\times\hat{p}_c)]\rangle$ is a scalar observable which is invariant under the parity reflection operation. The polarization component $P_\perp$ is thus present independent of the parity invariance properties of the beta interaction.

For allowed and first-forbidden ξ transitions, the β parameter $A_1(\beta)$ is proportional to $p/W=v/c$, where v is the velocity of the β particle. It is then convenient to write Eq. (26) in the form

$$P_\|(\theta)=K\ (v/c)\ [\sin\theta/W_c(\theta)],\quad(30)$$

where K is independent of the angle θ and of the energy of the β particle. $W_c(\theta)$ is the $\beta-e^-$ directional correlation. We refer to K as the polarization constant of the conversion electrons,

$$K=-A_1(\beta)C_{1,\|}(e^-)(c/v).\quad(31)$$

3. EXPERIMENTAL ARRANGEMENT

The transverse polarization of an electron beam can be determined by using the fact that the differential cross section of the elastic scattering of electrons in the Coulomb field of nuclei (Mott scattering) depends on the transverse polarization of the electrons, since the scattering force is spin-dependent.[14] An electron of velocity $\mathbf{v}$ with its spin "up" approaching a positively charged nucleus straight ahead is preferentially scattered to the right, because at a distance r from the nucleus it experiences a force $\mathbf{F}\propto\nabla\{\boldsymbol{\sigma}\cdot[\mathbf{v}\times\mathbf{E}(r)]\}$ by virtue of its magnetic moment $\boldsymbol{\mu}$ which interacts with the strongly varying magnetic field B [$\mathbf{B}$ (at electron) $=\mathbf{E}(r)\times\mathbf{v}$]. The differential scattering cross section can be written as

$$\begin{aligned}\frac{d\sigma}{d\Omega}(\hat{p},\hat{p}',P_T\mathbf{n})&=\left[1+a(\psi)P_T\mathbf{n}\cdot\frac{(\hat{p}\times\hat{p}')}{\sin\psi}\right]\frac{d\sigma_0}{d\Omega}(\psi)\\&=(1+a(\psi)P_T\cos\phi)\frac{d\sigma_0}{d\Omega}(\psi),\end{aligned}$$

where $\hat{p}$ and $\hat{p}'$ are unit vectors in the direction of the electron momentum before and after scattering. The angle ψ is the scattering angle, $\cos\psi=\hat{p}\cdot\hat{p}'$, and $a(\psi)$ is the asymmetry factor, which depends on ψ and on the energy of the electron.[15] The azimuth angle ϕ is the angle between the transverse polarization axis $\mathbf{n}$ and the normal $\mathbf{s}=\hat{p}\times\hat{p}'$ to the scattering plane.

Experimentally, the scattering asymmetry at a certain angle ψ,

$$\delta_\phi(\psi)=\frac{I_\phi(\psi)-I_{\phi+\pi}(\psi)}{I_\phi(\psi)+I_{\phi+\pi}(\psi)}=a(\psi)P_T\cos\phi\quad(32)$$

is observed. $I_\phi(\psi)$ is the intensity of the electron beam after scattering at an angle ψ whose original transverse polarization axis $\mathbf{n}$ was at an angle ϕ with respect to the normal $\mathbf{s}=\hat{p}\times\hat{p}'$ of the scattering plane. We refer to the direction $\mathbf{s}$ as the scattering axis.

For a maximum observable effect the scattering arrangement is usually designed in such a way that $\phi=0$; then,

$$\delta_0(\psi)=a(\psi)P_T.\quad(33)$$

In any experimental arrangement, however, a considerable spread of ϕ and ψ exists due to the finite solid angles of the detectors and the finite size of the electron beam. Proper corrections must be applied to extract P_T from the experimental data. Since the direction of the transverse polarization axis is determined by the direction Ω_1 of the momentum $\mathbf{p}_\beta$ of the preceding β particle, that is observed with a finite size detector, also a spread of the $\mathbf{n}$ direction must be taken into account in our measurements. The measured scattering asymmetry is, therefore, a weighted average

$$\bar{\delta}=\int f(\Omega_1\Omega_2)P_T(\theta(\Omega_1,\Omega_2))a(\psi(\Omega_1\Omega_2))\frac{d\sigma_0}{d\Omega}(\psi)d\Omega_1d\Omega_2\Big/\int f(\Omega_1\Omega_2)\frac{d\sigma_0}{d\Omega}(\psi)d\Omega_1d\Omega_2,\quad(34)$$

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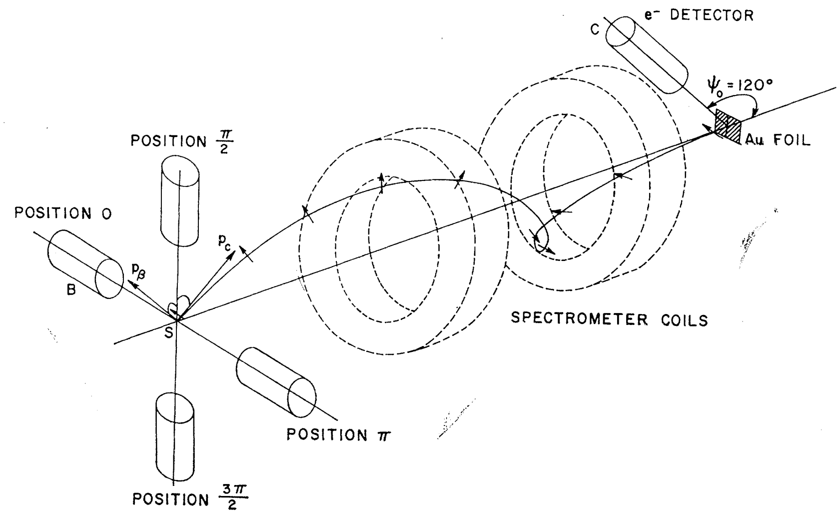

FIG. 2. Geometry of detector arrangement for $\beta - e^-$ polarization correlation measurements.

where Ω_2 describes the direction $\hat{p}'$ of the scattered electron. The function $f(\Omega_1\Omega_2)$ is a weighing factor which must be determined by the geometry of the experimental arrangement.

The experimental apparatus was designed to measure the transverse polarization $P_{\parallel}(\theta)$ in the plane of the beta and conversion electron momenta $\mathbf{p}_\beta$ and $\mathbf{p}_c$. A schematic diagram of the apparatus is shown in Fig. 2. The momentum direction of the β particles is defined by the beta-detector B which consists of a $\frac{1}{8}$-in. plastic scintillator disk mounted by means of a 1-in. light pipe to an RCA 6342 A photomultiplier.

The conversion electrons are focused by means of a double lens spectrometer onto a 0.62 mg/cm^2 gold scattering foil which is mounted perpendicularly to the axis of the spectrometer. The scattered electrons are detected by a scintillation counter C whose axis is at an angle $\psi_0 = 120°$ with respect to the spectrometer axis. The conversion electron detector C consists of an $\frac{1}{8}$-in. plastic scintillator mounted on a 3-in. light pipe which is viewed by an RCA 6342 A photomultiplier.

The conversion electron trajectories in the spectrometer were defined by an aluminum baffle near the source as shown in Fig. 3. A second aluminum baffle was placed at the ring focus of the lens to reduce the probability of scattered electrons entering the detector area. The scattered electron background was minimized by ten Plexiglas antiscattering baffles inserted between the two aluminum baffles. A lead core around which the baffle system was constructed prevented direct gamma radiation from reaching the foil. With the baffle configuration as described, the spectrometer resolution was 3.2%; the transmission was 2.6%.

The determination of the transverse polarization from Mott scattering requires the measurement of the intensity of the scattered electron beam to the left and to the right of the correlation plane (determined by axis of spectrometer and beta counter). In order to avoid any asymmetries at the scattering end of the spectrometer, the position of the C detector was not changed, rather the direction of the polarization vector of the conversion electrons was reversed by rotating the β detector (with the source) by π about the spectrometer axis. This method made it possible to determine any

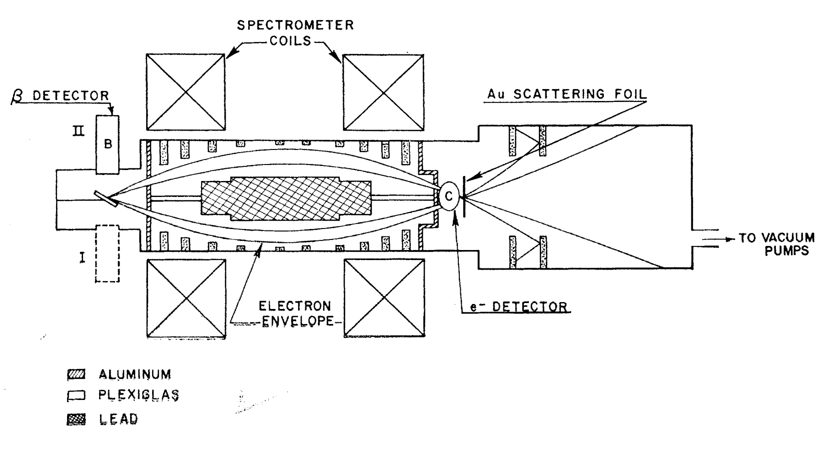

FIG. 3. Cross section of lens spectrometer and scattering foil arrangement.

solid-angle asymmetries due to the change of the position of the β detector at the source end of the spectrometer by measuring the (large) rate of $\beta-e^-$ coincidences with the scattering foil replaced by an e^- detector. Such a measurement does not provide a test for any asymmetry built into the instrument, e.g., the introduction of a preferred scattering direction due to the position of baffles, chamber walls, etc. Tests for those asymmetries are discussed later.

Behind the scattering foil, which was mounted on stretched nylon threads, a large vacuum chamber served as a catcher for the electrons that were not scattered by the gold foil. In this way, the detection of electrons scattered from other places than the gold foil was minimized. This was experimentally verified by performing measurements with the foil removed. The very small background observed with the foil removed was taken into account in the polarization measurements.

The coincidence electronics consisted of a fast-slow coincidence circuit with differential pulse-height analysis in both channels. The resolving time of the "fast" coincidence circuit was 35 nsec.

The use of a magnetic spectrometer in selecting the polarized conversion electrons requires careful consideration of the motion of the spin of the electrons in the magnetic field. The electrons move along a helical trajectory with the relativistic orbital "cyclotron" frequency $\omega_c = eB/mc$, where m is the relativistic mass of the electron and B is the (uniform) magnetic field [Fig. 4(a)]. The precession frequency of the spin of the electron about the direction of $\mathbf{B}$ is

$$\text{for } \mathbf{B} \text{ parallel to } \mathbf{p}_c, \quad \omega_s = \omega_c(1+a); \tag{35a}$$

$$\text{for } \mathbf{B} \text{ normal to } \mathbf{p}_c, \quad \omega_s = \omega_c[1+a/(1-\beta^2)^{1/2}]; \tag{35b}$$

where a is the anomalous part of the electron g factor: $a=(g-2)/2 \simeq \alpha/2\pi \approx 10^{-3}$. Considering the accuracy of our experiments (a few percent), the influence of a can be completely neglected, and we assume in the following $\omega_s = \omega_c$. Since the spin precession ω_s and the orbital frequency ω_c are the same, the spin precesses by 2π if the spectrometer trajectory reaches the spectrometer axis at the target point T (position of Mott-scattering foil) [Fig. 4(b)]. These considerations are correct for a homogeneous magnetic field parallel to the spectrometer axis. The magnetic field in the spectrometer used for the present measurements is not quite homogeneous. An exact calculation of the spin precession in the actual magnetic field of the spectrometer showed that the above mentioned conclusions are correct within a few percent. The transverse polarization of the electron beam with respect to the scattering axis [x' axis in Fig. 4(c)] as it reaches the target via different spectrometer trajectories is smeared out due to four effects: (1) The finite size of the β detector and of the source causes a spread in the direction of $\mathbf{p}_\beta$; (2) Scattering in the source and in its environment reduces the polarization of the conversion electrons; (3) Due to the spherical aberration of the lens spectrometer, the electrons which hit the target T follow, in general, trajectories which cross the spectrometer axis slightly in front or behind the target. The precession angle of the spin of electrons on those trajectories is larger or smaller than 360°. The total spread of the spin precession angles in our arrangement is from 342 to 370°; (4) Due to the finite, annular entrance aperture of the spectrometer, the polarization (with respect to the scattering axis x') of a conversion electron with initial momentum $\mathbf{p}_{c0}$ in the $z-y$ plane [e.g., trajectory 1 in Fig. 4(c)] is different from the polarization of an electron which has its initial momentum in the $x-z$ plane [e.g., trajectory 2 in Fig. 4(c)], although the β particles preceding the emission of the two conversion electrons were observed in the same direction $\mathbf{p}_{\beta 0}$. We assume in the following that only the transverse polarization $P_{\text{II}}(\theta)$ is different from zero, i.e., that $\mathbf{n}$ is in the direction $\mathbf{p}_{c0}\times(\mathbf{p}_{\beta 0}\times\mathbf{p}_{c0})$. We also assume that $A_3(\beta)$ can be neglected. It will be shown later that these two conditions are satisfied for the case under investigation. The transverse polarization of the conversion electrons is thus $P_T(\theta) = P_{\text{II}}(\theta) = K(v/c)[\sin\theta/W(\theta)]$. An electron following trajectory 1 of Fig. 4(c) has a transverse polarization $P_T(90°) = K(v/c)[1/W(90°)]$ and $\mathbf{n}$ is parallel to $\mathbf{p}_{\beta 0}$, which we chose parallel to the x axis. Such an electron, after a 360° spin precession, reaches the scattering foil possessing a transverse polarization P_s with respect to the scattering axis of

$$P_s = P_T(90°) = K\ (v/c)\ [1/W(90°)].$$

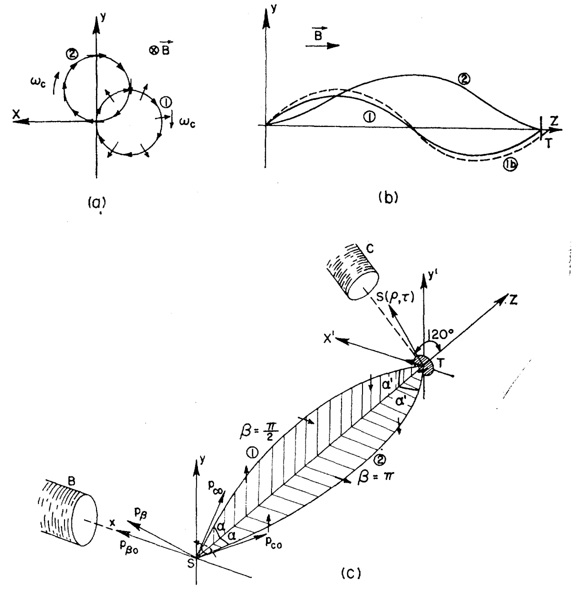

FIG. 4. The "cyclotron" and "spin" motion of electrons in the β-ray spectrometer. (a) Looking along the axis of the spectrometer; (b) side view of the same trajectories; (c) perspective view with cyclotron motion omitted.

(We assumed a trajectory that crossed the spectrometer axis at T.) On the other hand, an electron following the trajectory 2 of Fig. 4(c) has a transverse polarization $P_T(90°+\alpha)=K(v/c)[\cos\alpha/W(90°+\alpha)]$ and its transverse polarization axis $\mathbf{n}$ is at an angle α with respect to $\mathbf{p}_{\beta 0}$ and the x axis. This electron reaches the scattering foil with its polarization vector $\mathbf{n}$ at an angle α' with respect to the scattering axis. The transverse polarization of such an electron is $P_s=K[\cos\alpha\cos\alpha'/W(90°+\alpha)]$ with respect to the scattering axis. Since this effect depends on the azimuth angle β of the $\mathbf{p}_{c0}-z$ plane we call it the azimuth effect.

Taking into account all these four effects, the average polarization $P_s(\alpha,\beta)=K(v/c)f(\alpha,\beta)$ of a conversion electron with respect to the scattering axis was computed for various angles β and α ($\alpha_{\min}=12.3°$, $\alpha_{\max}=20.5°$).

Since the β detector accepts β particles within a certain energy range, the polarization $P_s(\alpha,\beta)$ must be averaged over v/c. In this way we obtain $P_s(\alpha,\beta)=K\langle v/c\rangle_{\text{av}}f(\alpha,\beta)$.

In the scattering process two effects must be considered: (1) The reduction of the effective asymmetry parameter $a(\psi)$ from the theoretical value $a(\psi)$ due to plural and multiple scattering in the gold foil of finite thickness must be taken into account. This effect was computed on the basis of the procedure discussed by Wegener[16]; (2) The spread of the scattering angle ψ causes a reduction of the observed counting asymmetry. The angle ψ depends on the polar angles α' and β' which describe the direction in which the conversion electron beam strikes the scattering foil. Since we have already computed the polarization $P_s(\alpha,\beta)$ with respect to the scattering axis x', we replace α' and β' [which are defined in the same way as α and β but with respect to the x' and y' axes, see Fig. 4(c)] by $\alpha+\pi$ and β, respectively. This approximation introduces only a small error (of order 2%). The scattering angle ψ depends, of course, also on the direction (ρ,τ) of the scattered beam, which is detected by the finite-size conversion electron detector C. The polar angles (ρ,τ) are defined with respect to the axis of detector C.

We obtain thus for the observed counting rate asymmetry

$$\bar{\delta}=\frac{N_0-N_\pi}{N_0+N_\pi}=K\left\langle\frac{v}{c}\right\rangle_{\text{av}} \times\left[\int f(\alpha\beta)\bar{a}(\psi(\alpha\beta\rho\tau))\frac{d\sigma_0}{d\Omega}\sin\alpha\, d\alpha d\beta\sin\rho\, d\rho d\tau \Big/ \int f(\alpha,\beta)\frac{d\sigma_0}{d\Omega}\sin\alpha\, d\alpha d\beta\sin\rho\, d\rho d\tau\right], \quad (36)$$

or

$$\bar{\delta}=K\langle v/c\rangle_{\text{av}}R(E_c), \quad (37)$$

where N_+ and N_- are the coincidence counting rates observed with the β counter in position 0 and position π, respectively (see Fig. 2). The polarization figure of merit $R(E_c)$ of the whole experimental arrangement depends, of course, on the energy E_c of the conversion electrons.

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4. POLARIZATION OF THE Hg^{203} *K*-CONVERSION ELECTRONS

4.1. Decay of Hg^{203}

The decay of Hg^{203} is shown in Fig. 5. The β transition is first forbidden with a logft value of 6.4. Within statistical error, the shape of the Hg^{203} β spectrum is of the allowed form, a fact which suggests that the ξ approximation is valid for the description of this β transition. The γ transition is a mixed $M1+E2$ multipole transition with an amplitude mixing ratio of $\delta(\gamma)=\langle\frac{3}{2}\|E2\|\frac{1}{2}\rangle/\langle\frac{3}{2}\|M1\|\frac{1}{2}\rangle=+1.5$. The amplitude mixing ratio for K-conversion electrons is $\delta(e_k^-)=+0.64$. The $\beta-\gamma$ directional correlation exhibits a very small anisotropy[17]:

$$W_{\beta\gamma}(\theta)=1-(0.0022\pm0.0009)P_2(\cos\theta).$$

This is a further indication of the validity of the ξ approximation for the Hg^{203} β decay. From the experimental directional correlation factor $A_2(\beta)A_2(\gamma)=-0.0022\pm0.0009$, the β parameter $A_2(\beta)$ can be computed: $A_2(\beta)=+0.002\pm0.001$. With this value the transverse polarization of the conversion electrons perpendicular to the $\beta-e_k^-$ plane can be computed from Eq. (27) using the expressions of Becker and Rose for the polarization particle parameters $b_{2\perp}(L\pi L'\pi')$. The computed value of the transverse polarization of the conversion electrons perpendicular to the $\mathbf{p}_\beta-\mathbf{p}_c$ plane $P_\perp(\theta)$ is smaller than 0.001. This justifies, at least for Hg^{203}, the neglect of this type of polarization in our earlier discussion of the polarization figure of merit of the spectrometer.

In the same way, it can be shown that the $\beta-e_K^-$ directional correlation, which has an anisotropy of less than 0.003, can be neglected for Hg^{203}. Thus, within our error limits, we can write for the transverse polarization $P_{\parallel}(\theta)$ parallel to the $\mathbf{p}_\beta-\mathbf{p}_c$ plane of the Hg^{203} conversion electrons:

$$P_{\parallel}(\theta)\cong K(v/c)\sin\theta. \quad (38)$$

The angular momentum I_i of the Hg^{203} ground state is

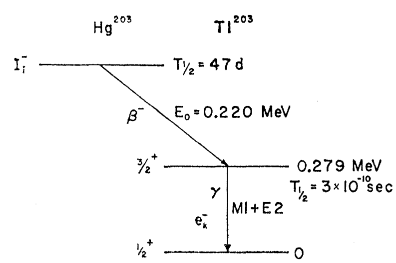

FIG. 5. Decay of Hg^{203}.

[17] R. M. Steffen (unpublished).

not known. On the basis of the β-decay data, the values $I_i=\frac{1}{2}$, $I_i=\frac{3}{2}$ and $I_i=\frac{5}{2}$ are possible. These three values give for $A_1(\beta)$, according to Eq. (10),

$$I_i=\tfrac{1}{2} \quad A_1(\beta)=+\tfrac{2}{3}F_1(11\,\tfrac{1}{2}\,\tfrac{3}{2})\frac{v}{c}=-0.745\frac{v}{c}; \tag{39a}$$

$$I_i=\tfrac{3}{2} \quad A_1(\beta)=+\frac{2}{3}\,\frac{\sqrt{3}y'-0.447y'^2}{1+y'^2}\,\frac{v}{c}; \tag{39b}$$

$$I_i=\tfrac{5}{2} \quad A_1(\beta)=+\tfrac{2}{3}F_1(11\,\tfrac{5}{2}\,\tfrac{3}{2})\frac{v}{c}=+0.447\frac{v}{c}; \tag{39c}$$

where we have assumed the validity of the ξ approximation. The matrix-element parameter y' is given in Eq. (12).

The polarization factor $C_{1,\text{II}}(e^-)$ of the Hg^{203} conversion electrons is given by Eq. (28). We use the particle parameters $b_{1,\text{II}}(L_c,\pi)$ of Becker and Rose and obtain $C_{1,\text{II}}(e^-)=0.73$. The polarization constant K of Eq. (38) for the three spin possibilities is, therefore, [cf. Eq. (31)]

$$I_i=\tfrac{1}{2} \quad K(\tfrac{1}{2})=+0.54, \tag{40a}$$

$$I_i=\tfrac{3}{2} \quad K(\tfrac{3}{2})=\frac{-0.84y'+0.22y'^2}{1+y'^2}, \tag{40b}$$

$$I_i=\tfrac{5}{2}, \quad K(\tfrac{5}{2})=-0.33. \tag{40c}$$

The experimental determination of the polarization constant K makes it possible to choose between these three possibilities.

4.2. Preparation of the Hg^{203} Source

The preparation of the Hg^{203} sources used in the measurements required special precautions to avoid contamination of the instrument. The radioactive Hg^{203} was supplied by the Oak Ridge National Laboratory in the form of $Hg^{203}(NO_3)_2$. The nitrate was reduced to metallic Hg which was deposited on a $\frac{1}{4}$-mil Mylar foil as a thin uniform film. The thickness of the source was of the order of 10 μg/cm^2. The Mylar foil was supported by a very thin aluminum ring of 1-in. diameter.

Because of the relatively high vapor pressure of Hg at room temperature, it was necessary that the Hg be positively isolated from the spectrometer vacuum system. This was achieved by covering the source with a second $\frac{1}{4}$-mil Mylar foil and bonding the Mylar to the source holder ring under vacuum. A detailed description of this technique will be published elsewhere.

4.3. Measurements and Result

The measurement of the transverse polarization of the Hg^{203} conversion electrons was done by recording the coincidence counting rate of the β particles registered in the β detector B (Fig. 2) and the scattered conversion electrons registered in detector C. The difference in the coincidence counting rates C_0 and C_π with the beta detector in positions 0 and π was determined and the asymmetry $\bar{\delta}$ [see Eq. (36)] was computed from the corrected coincidence counting rates N_0 and N_π. The position of the β detector was changed in intervals of 15 min.

The β detector accepted β particles in the energy interval from 0.080 MeV to 0.210 MeV, corresponding to an average value of v/c of $\langle v/c\rangle_{\text{av}}=0.59\pm0.02$.

The result of the six-months measuring period can be summarized as

$$\bar{\delta}=(N_0-N_\pi)/(N_0+N_\pi)=+0.079\pm0.007. \tag{41}$$

From the computed polarization figure of merit $R(E_c)$ for the Hg^{203} K-conversion electrons R(0.193 MeV)$=0.215+0.008$, the polarization factor K was computed from (37) with the result

$$K(e_k^-,\text{Hg}^{203})=+0.62\pm0.08. \tag{42}$$

The positive sign of $K(e_k,\text{Hg}^{203})$ indicates that the polarization vector of the Hg^{203} conversion electrons points in the same direction as the momentum of the preceding β particle.

The transverse polarization of the Hg^{203} conversion electrons parallel to the $\mathbf{p}_\beta-\mathbf{p}_c$ plane is thus given by

$$P(\theta)=+(0.62\pm0.08)(v/c)\sin\theta. \tag{43}$$

This experimental value of $K(e_k,\text{Hg}^{203})$ confirms the positive sign of the polarization as reported by Alberghini and Steffen,[6] Blake *et al.*,[8] and recently by Bhattacherjee *et al.*[18] The magnitude of the polarization is considerably larger than reported by Alberghini and Steffen,[6] who did not use a spectrometer for the selection of the conversion electrons and whose measured value of K might, therefore, be affected by scattering effects. Our value of K is also considerably larger than the value $K=+0.30\pm0.08$ reported by Blake *et al.*,[8] but it is in excellent agreement with the value $K=+0.66\pm0.10$ of Bhattacherjee *et al.*[18]

It should be noted that all the measurements mentioned above were evaluated on the basis of the theoretical values of the asymmetry parameter $a(\psi)$ as computed by Sherman.[15] These calculations were made for point nuclei and without taking into account the screening due to the presence of atomic electrons. The finite size of the nucleus is probably not important for the electron energies involved in these experiments.[19] It is known, however, that screening corrections are important for electron energies below 0.2 MeV.[20–22]

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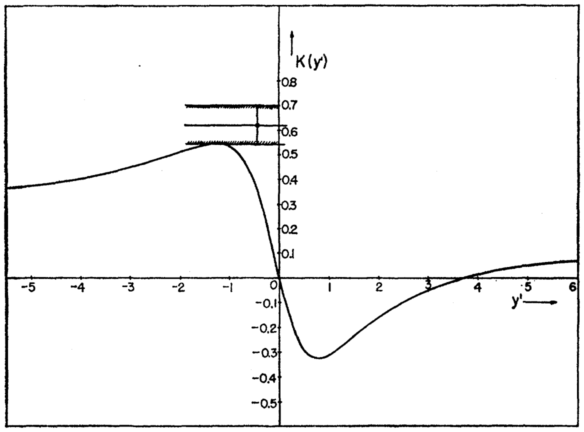

FIG. 6. Polarization constant $K(\frac{3}{2})$ as a function of the matrix element parameter y'.

Some calculations of $a(\psi)$ using screened Coulomb wave functions have been made, but their accuracy is limited.[23,24]

4.4. Discussion

The experimental value of $K(e_k, \mathrm{Hg}^{203})$ excludes the spin assignment $I_i = \frac{5}{2}$ to the Hg^{203} ground state [see Eqs. (40)]. The spin assignment $I_i = \frac{1}{2}$ is compatible with our result, although it is somewhat less likely than the $I_i = \frac{3}{2}$ spin assignment in view of the absence of a β transition to the Tl^{203} ground state of spin $I_f = \frac{1}{2}$.

The spin assignment $I_i = \frac{3}{2}$ to the Hg^{203} ground state implies the possibility that both scalar- and vector-type beta matrix elements contribute to the beta decay. Figure 6 shows the polarization constant $K(\frac{3}{2})$ as a function of the matrix element ratio y' of Eq. (12). The experimental value of K yields the value of y',

$$y' = -1.25 \pm 0.15,$$

if the $I_i = \frac{3}{2}$ spin assignment is accepted.

With this value of y', the expected $\beta-\gamma$ circular polarization correlation can be computed from Eqs. (10) and (15). Since $A_2(\beta) \simeq 0$, one obtains

$$P_c(\theta)_{\mathrm{Hg}^{203}} = A\,(v/c)\cos\theta,$$

with

$$A = +0.33 \pm 0.04.$$

The $\beta-\gamma$ circular polarization anisotropy A of Hg^{203} has been measured by Boehm and Wapstra.[25] Their experimental value $A_{\mathrm{exp}} = -0.06 + 0.22$ does not agree too well with the results of the conversion-electron polarization measurements.

ACKNOWLEDGMENTS

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